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TO FIND THE DIFFERENTIAL OF A VARIABLE QUANTITY WITHOUT THE USE OF INFINITESIMALS OR LIMITS.

BY PROF. JOSEPH FICKLIN, COLUMBIA, MISSOURI.

I. If any function increases at a uniformly accelerated rate, the increment of that function in one unit of time is equal to the rate at which that function increases at the middle of that unit of time.

II. Let $u = x^2$, and suppose dx to be the *rate* at which x varies; then

$u' - u = 2xdx + dx^2$ = increment of u in first unit of time;
$u'' - u' = 2xdx + 3dx^2$ = " " " " sec'd " " " ;
$u''' - u'' = 2xdx + 5dx^2$ = " " " " third " " " ;
$u'''' - u''' = 2xdx + 7dx^2$ = " " " " fo'rth " " " ;

. . . .

Hence u increases faster each instant; and, since there is a constant difference ($2dx^2$) in the increments of u, the *acceleration* in the *rate* of increase of u is uniform.

Now, since the difference between u' and u is $2xdx + dx^2$, and, since u is increasing at a *uniformly accelerated rate,*

$$2xdx + dx^2$$

is not only the *mean* rate at which u increases during the first unit of time, but u increases *at that rate* at the *middle* of that unit of time (I).

But the *rate* at which u increases is increased by $2dx^2$ in one unit of time, for u is increasing at the rate of $2xdx + dx^2$ at the middle of the first unit, and at the rate of $2xdx + 3dx^2$ at the middle of the second unit; hence, in half a unit of time, the rate is increased by dx^2. Therefore, if du represents the *rate* at which u increases at the instant $u = x^2$, then

$$du + dx^2$$

will be the rate at which u increases at the middle of the first unit of time.

$$\therefore \qquad du + dx^2 = 2xdx + dx^2;$$

whence,

$$du = 2xdx.$$

The same result may be obtained by equating the two expressions for the rate at which u increases at the middle of any other unit of time. Thus,

$$2xdx + 5dx^2$$

is the rate at which u increases at the middle of the third unit of time, and, since the rate at which u increases is increased by $2dx^2$ in one unit,

$$du + 2\tfrac{1}{2} \times 2dx^2$$

is also an expression for that rate;

$$\therefore \qquad du + 2\tfrac{1}{2} \times 2dx^2 = 2xdx + 5dx^2;$$

whence,

$$du = 2xdx.$$

III. Let $u = xy$; then

$$\begin{aligned} u' - u &= xdy + ydx + dxdy; \\ u'' - u' &= xdy + ydx + 3dxdy; \\ u''' - u'' &= xdy + ydx + 5dxdy; \\ u'''' - u''' &= xdy + ydx + 7dxdy; \end{aligned}$$

. . . .

In this case the constant difference in the increments of u is $2dxdy$; and it may be shown, by a process of reasoning entirely similar to that used in II, that

$$xdy + ydx + dxdy$$

is the rate at which u increases at the middle of the first unit of time, and that

$$du + dxdy$$

is also an expression for that rate;

$$\therefore \qquad du + dxdy = xdy + ydx + dxdy;$$

whence,

$$du = xdy + ydx.$$

If we put $y = x^2$, we have

$$du = d(xy) = d(x^3) = xd(x^2) + x^2dx.$$

But it has been shown that $d(x^2) = 2xdx$;

$$\therefore \qquad du = d(x^3) = x \times 2xdx + x^2dx = 2x^2dx + x^2dx = 3x^2dx.$$

In this way it may be shown that, if n be any positive integer,

$$du = d(x^n) = nx^{n-1}dx.$$

It may now be shown, in the usual way, that this equation is true, whether n be positive or negative, integral or fractional.

THE METEOR OF AUGUST 11, 1878.

BY PROFESSOR DANIEL KIRKWOOD.

1. *Observations at Bloomington, Ind.*—A few minutes after 10 o'clock* on Sunday evening, Aug. 11, 1878, Rev. John A. Bower of Bloomington, Ind., saw a brilliant meteor near the eastern horizon. Mr. B. had just taken

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*The *precise* time was not noted.